Unraveling the Size Effect

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The widely accepted capital asset pricing model (henceforth CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) postulates a simple linear relationship between a stock's expected return and its risk. However, recent evidence has shown that other factors have a consistent and significant effect on common stock return. Basu (1977) finds that price-earnings ratios and risk adjusted returns are related. A study performed by Litzenberger and Ramaswamy (1979) shows a significant positive relationship between dividend yield and returns on common stock. One of the most discussed relationships, and the main focus of this study, is the one between a company's size and the return on its stock. This anomaly, now known as the size effect, has been the focus of recent studies conducted by Fama and French (1992) as well as Daniel and Titman (1997), however the seminal work was performed by Banz in 1981. His findings show that the size of a firm and the return on its common stock are inversely related. While Banz's findings were shown to be accurate and his models appear to address the possible econometric problems involved, he can not offer a theoretical foundation for this relationship. Therefore, Banz suggests that size may be a proxy for other factors that were not tested but are correlated to size.

It is for this reason that a second test is conducted in this study. The first test is performed to verify the existence of the size effect in the sample collected. The second test will use dummy variables to distinguish between large and small firms. Dummy interaction variables will then be used to capture the difference in how large and small firms are affected by other factors. If the dummy interaction variables show a remarkable difference in the way small firms are affected by a certain factor when compared to large firms, then a step has been taken towards explaining the anomaly known as the size effect. Banz's research and other relevant works are reviewed in section II. Section III defines the data that was used for this experiment. The first model, which tests for the existence of the size effect in the sample, is presented in section IV. The second test is presented in section V with the results interpreted in section VI. Finally, section VII offers a conclusion to the tests performed and suggestions for further research.

I. Theory

The existence of the size effect has some specific implications for both the CAPM and the efficient market hypothesis. The CAPM assumes that the expected return from an asset is a function of its price variance. This figure is usually reported as beta and is synonymous with risk. This relationship is thought to be linear and positive, hence the adage "high risk, high return". Several assumptions were made by Sharpe (1964), Lintner (1965) and Mossin (1966) when they developed the CAPM. First they assume that an investors portfolio will maintain a constant proportion between risky and risk-free assets. A second assumption is that all investors can lend or borrow money at the risk-free rate.

Assuming these things to be true, they devised the following equation:

$$\mu_j - r = \theta_m\sigma_{jm}$$

Where:

μ_j = expected return on any asset *j*
r = the risk free rate
σ_{jm} = covariance of stock return with return on the market portfolio (AKA beta)
θ_m = measure of aggregate risk aversion

To avoid confusion beta will be spelled out when referring to stock risk and the symbol b will be used when it represents the estimated parameters in the regression functions.

This equation states that any return that exceeds the risk free rate, also known as risk premium, will be proportional to that stock's beta. Since beta and risk aversion are the only variables on the right hand side of the equation, any theory that suggests another factor consistently affects return would require the rejection of the CAPM.

A more established theory known as the efficient market hypothesis also conflicts with Banz's findings. A capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Thus it is impossible to make economic profits by trading on the basis of such information. This is implied because people are assumed to be rational. An indication of abnormally high profits will attract investors and increase the demand for that security. In turn the price for that security will increase eliminating excess profits. Since the size of a company is public information, buying stocks on the basis of firm size should not lead to higher returns. However, Banz's 1981 study indicates otherwise.

The sample used for Banz's research spanned 50 years from 1925 through 1976. During this time any stock that was traded on the NYSE for at least five years was included in the sample. Banz's model assumed that in addition to risk, stock returns were a function of firm size. The model used took this form:

$$E(R_i) = \beta_1 + \beta_2 \text{ (beta)} + \beta_3 (\phi_i + \phi_m) / \phi_m$$

Where:

$E(R_i)$ = expected return on asset *i*
β_1 = expected return on asset with beta of zero (Y intercept)
β_2 = expected market risk premium
beta = measurement of risk
β_3 = constant measuring the contribution of size to the expected return on an asset
ϕ_i = market value of security *i*
ϕ_m = average market value

Banz used total market value of outstanding common stock to represent firm size. If there was no relationship between firm size and return on common stock, then β_3 would equal zero and Banz's equation would be reduced to the CAPM in which return is proportionally related to risk alone.

Banz takes several approaches to testing this equation. One in particular seems to eliminate most econometric problems and yields the most reliable results. First, the companies are split into five portfolios depending on size. Those portfolios are then split five more times according to each stocks beta. The result is 25 portfolios each containing a similar number of securities based on size and risk criteria. Banz reports a significant and negative parameter for size, thus indicating that firms with large market values have smaller returns than small firms with comparable beta figures. Again, Banz does not offer a theoretical explanation for the size effect. What is significant about his study is that it contradicts two widely accepted theories, the CAPM and the efficient market hypothesis.

II. Data

This regression will use pooled cross sectional data taken from the 1999 Value Line Investment Survey. Over 1500 companies are listed, however the nature of the tests prevented the use of all 1500. The figures chosen for the regression are those that cover a span of ten years, from 1989 through 1998. As a result survivorship bias is present throughout the experiment, that is only firms that have been trading for at least ten years and were still being traded at the end of 1998 can be included in the regression. After eliminating the ineligible companies the sample size is reduced to 1159.

There are a number of ways to determine the size of a company, Banz chose to use total market value of common stock. This experiment will utilize total assets to compare results. Both of these figures have their limitations. Market value is simply the price of a stock multiplied by the number of outstanding shares. As a result it is easily inflated or deflated by trends in the market. Considering that firms often have millions upon millions of shares outstanding, even a one point change in stock price can have a remarkable effect on the market value of that firm. At the same time the reported figure for total assets relies on book values which are in turn dictated by somewhat arbitrary accounting rules. Of course, each stocks beta is included to account for the risk premium. Net income is included in the regression because, as a component of retained earnings, it has a direct bearing on the value of common stock. Earnings per share is included in the regression because it is an indication of how the earnings of a company are distributed among its owners. Basu (1977) suggests a relationship between a company's price-earnings ratio and return on their stock. This ratio indicates how much one dollar of a company's earnings will cost an investor in the form of common stock. An increase in this ratio would imply that more investor capital is required by the company to earn a dollar. These figures will be regressed on ten year return. This figure represents what an investor would realize in capital gains after holding on to the stock for the full ten years.

III. Model 1

This model is designed to test for the existence of the size effect in the sample collected. All of the variables are included in order to avoid misspecification bias, however the only parameter of concern is that of the total assets variable. Ordinary least squares regression is performed on the following function:

$$R = \beta_1 + \beta_2 (NI) + \beta_3 (TA) + \beta_4 (EPS) + \beta_5 (P/E) + \beta_6 (beta)$$

Where:

R = ten year return
β_i = estimated parameter for the given variable
NI = Net Income
TA = Total Assets
EPS = Earnings per Share
P/E = Price-Earnings Ratio
beta = measurement of risk

The results for the OLS regression are presented in table 1.

Table 1

Variable	Parameter	Standard Error	T for H_o	Prob > T	
Intercept	3.879052	0.91869733	4.222	0.0001	**R Square:** .1114
Net Income	0.003475	0.00045949	7.563	0.0001	**Adjusted R Square:** .1083
Total Assets	-0.000021	0.00000876	-2.588	0.0100	
EPS	0.574468	0.21988731	6.613	0.0091	**F Value:** 36.2
P/E	0.197713	0.03903474	5.065	0.0001	
Beta	6.280939	0.81119588	7.743	0.0001	**Prob > F:** .0001

All of the tested variables are shown to be significant on at least a 90% confidence interval. We can reject the null hypothesis that these variables have no effect on return individually by way of the t-stats and collectively by observing the F-stat. Notice the parameter for total assets, though it is small it is also negative. This indicates that as the observed firms increased their total assets, the return on their common stock decreased.

Therefore it is clear that the size effect is at work in this sample and the second test can be performed in an attempt to explain it.

IV. Model 2

The second model utilized dummy variables to distinguish between the large and small companies. The companies were arranged in ascending order according to total assets. Then the middle 300 companies were dropped (a similar method was employed by Chatterjee and Maniam, 1997). Of the small firms in the sample, none had assets totaling more than $1.233 billion. Incidentally, the upper limit placed on small-cap companies by financial institutions is $1.5 billion. Total assets for the large firms began at $4.289 billion. By eliminating the middle 300 companies a $3 billion gap was created in the sample. This accomplishes two things. First, it ensures that there is a significant difference between the two size classes. Secondly, this is the most efficient way to reduce crossover bias. If the separation were a finite amount, firms would be crossing over that boundary repeatedly throughout the sample period, affecting the results of the regression. However, it is much less likely that a company will make the $3 billion jump.

The firms were then assigned dummy variables; the small firms assumed the value of unity and the large firms, zero. Dummy interaction variables were then created for each of the independent variables. The parameters for these variables will capture the difference between the large and small firms. They will indicate how the return on a small firm's stock would react, relative to that of a large firm, to a change in the given independent variable. By observing these it may be possible to back up the size effect with some theoretical foundation based in the financial figures.

The first regression utilized a linear model. However, scatter plots indicated heteroskedasticity in the net income variable. This was then confirmed by a Goldfeld-Quandt test. A generalized least squares regression was run using a log-log model. This accomplished two things at once. First, the heteroskedasticity was corrected and second, the t-values improved. This indicates that there were some non-linearities present in the sample and according to the law of diminishing returns there should be. For example, as a company's net income increases the stock will generally offer a greater return, however the returns will begin to trail off even if the net income figure does not. This would result in a non-linear relationship. The improved results did come at a price. Since logs were used, companies that reported a negative figure for one of the variables used had to be eliminated. The results from the GLS regression are presented in table 2.

Table 2

Variable	Parameter	Standard Error	T for Ho	Prob > T	
Intercept	0.540351	0.27774415	1.946	0.0520	**R Square:** .2564
ln(Net Income)**	0.252126	0.03793818	6.646	0.0001	**Adjusted R Square:** .2467
ln(EPS)*	-0.122607	0.06026329	-2.035	0.0423	
ln(P/E)*	0.174500	0.08073832	2.161	0.0310	**F Value:** 26.401
ln(Beta)*	0.173791	0.07502713	2.316	0.0208	
Dummy**	-1.595748	0.41607945	-3.385	0.0001	**Prob > F:** .0001
ln(Net Income)(D)	-0.038862	0.06994211	-0.638	0.5239	
ln(EPS)(D)**	0.555833	0.10374241	5.358	0.0001	
Ln(P/E)(D)**	0.714021	0.12137652	5.883	0.0001	
Ln(Beta)(D)	-0.130133	0.10952340	-1.188	0.2352	

* indicates a confidence level of at least 90%
** indicates a confidence level of at least 99%

SRF: % Return: .541 + .252ln(net income) - .122ln(EPS) + .174lb(beta) + .174ln(P/E) -.159ln(Dummy)(D) - .039ln(net income)(D) + .556ln(EPS)(D) -.13ln(beta)(D) + .714ln(P/E)(D)

(a partial F-test was performed on the variables shown to be insignificant and found the same to be true)

V. Interpretation of Final Results

All but two of the variables are shown to be statistically different from zero. Since natural logs were used each of the parameters indicates a stock return’s sensitivity to a 1% change in each of the independent variables over a ten year period. The parameters for the dummy interaction variables are difficult to interpret if observed alone. However, when compared to the relative variable for large firms we will be able to see how each size class is affected by the financial figures.

The parameter for the dummy variable serves as the y-intercept for the small companies. A parameter of -1.595 shows that the y-intercept would fall far below the one for large companies. The parameter for net income is 0.252, this indicates that a 1% increase in net income would yield a 0.25% increase in return. The parameter for the net income interaction variable is approximately -0.04. Since this variable captures the difference between the size classes, the data shows that the same event would yield a 0.21% increase in return on a small company’s stock. However, since the t-statistic is .5239, the evidence supporting this relationship is extremely weak. The data shows that a 1% increase in earnings per share would actually decrease the ten year return on a large firm’s stock by 0.12%. However, the same increase in a small company’s earnings per share would lead to an increase in return of 0.43%. This is the first of several interesting results of this experiment. The parameters for the P/E ratio reveal that a 1% increase will result in a 0.18% increase in return for large companies but a 0.89% increase on an investment in a small company. As the beta figure increases by 1% for a large company the return will increase by 0.17% and a small company’s stock would yield an increased return of 0.04%. This would suggest that the risk premium is less on investments made in smaller firms.

VI. Conclusion

The results from the first model are consistent with Banz's findings, the parameter for company size is negative indicating an inverse relationship between size and stock return. The fact that Banz used a different measure of firm size seems to have made little difference. The results from the second model may provide some insight to a theoretical foundation for the size effect. The variables that reported the largest differences between the size classes are those that have earnings in their calculation, EPS and P/E. Naturally, the distribution of earnings should be considered when attempting to explain the size effect.

Small companies are more concerned with building equity and gaining market share than large companies are. As a result their earnings are distributed differently. A small company is more likely to reinvest its earnings back into the company. Causing the retained earnings to grow faster and increasing the value of the common stock. However, a large company is more likely to use its earnings in ways that, generally, do not increase the value of its common stock. Paying dividends to preferred stockholders is one example. Transportation by helicopter or private jet for executives may be another. Since large companies are retaining a smaller percentage of their earnings than the small firms, the common stock is returning less to its owners.

This is one possibility supported by concrete data and empirical analysis. There are others. Perhaps the earnings per share and price-earnings ratio are serving as a proxy for other variables not yet tested. The most likely solution is that there are several contributing factors and earnings distribution may only be one of them. This factor can be analyzed further to see how much of the size effect it accounts for. Once that is established other theories can be approached in a similar manner in hopes of fully explaining the size effect.

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